Path Planning and Control for Multiple Point Surveillance by an Unmanned Aircraft in Wind (Draft)

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Abstract— **In this paper, we explore the surveillance of multiple waypoints by a constant velocity aircraft in the presence of wind. It is assumed that the aircraft has a maximum turning rate and that the wind is equal to a known constant plus small possibly time varying components. The proposed strategy consists of separate path planning and control algorithms. The path planning is done by calculating the shortest time path through all of the waypoints in the presence of a known constant wind. During this step, the allowed turning rate is assumed to be less than the actual maximum turning rate of the vehicle. This algorithm produces a ground path that can be tracked by the control algorithm. The control algorithm breaks the desired trajectory into smaller sections which can each be approximated by a polynomial. A spatial sliding surface controller is then used to track each polynomial in the presence of the unknown wind disturbances.**

I. INTRODUCTION

Recent advances in sensor and vehicle technology have made unmanned aerial vehicles (UAVs) a valuable new tool for a variety of applications including search and surveillance. By allowing these tools to be utilized more effectively, research in path planning and trajectory tracking for UAVs has the potential for real world benefit. This paper studies one such problem of path planning and trajectory tracking for the surveillance of multiple locations in the presence of wind.

A. Related Work

The problem of path planning for mobile robots has been widely studied in recent years. One commonly cited work by Dubins solves the problem of travelling from an initial position and orientation in the plane to a final position and orientation with a bounded curvature [11]. These results were later reproduced using optimal control methods [4], and the synthesis of the paths was further characterized [5]. These results have also been applied in higher level path planning algorithms to visit multiple sites in the absence of wind [25], [26] and to navigate around obstacles [1], [3]. Other work has studied coordinated rendezvous of multiple unmanned aerial vehicles [2]. The choice of the order to visit multiple points has also been widely studied as the well known travelling salesman problem. Recent results have studied this problem for vehicles with kinematic constraints [23], [21].

This work was supported by ONR grant #N00014-03-C-0187, SPO #016671-004, and a Berkeley Fellowship for Mr. McGee

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Since the ability to follow desired paths is essential to any automated vehicle, the control problem of tracking a path has also been widely studied. One common approach is to represent the desired path with a time based trajectory which can be tracked using a variety of nonlinear controllers [12], [22] including sliding surface controllers [20], [27], [7], [6]. This use of time varying trajectories in the two dimensional plane requires at least two control inputs for the vehicle, typically a velocity control and a steering control. This problem is often addressed by decoupling the steering of the vehicle and its forward velocity control. One approach is to steer a vehicle to the desired path by defining polynomial curves which are tangent to both the vehicle velocity vector and the path [13]. A variety of sliding surface controllers have also been designed to steer along gradient lines of a potential field [14] or to drive the lateral distance to the reference path to zero using linearized models [15], nonlinear coordinate transformations [16], and reference heading angles calculated by the normal distance to the path [9]. Several researchers have used spatial dynamics of the vehicle to design controllers. One of these works introduces a spatial Laplace operator to design a linear controller based on a linearized model of the vehicle [8]. Other spatial approaches include spatial integration for a PI controller [10] and the use of feedback linearization on the spatial dynamics of the vehicle [18].

B. Main Contributions

The contribution of this work is the design of a complete system including both path planning and control algorithms for the surveillance of multiple locations in the presence of wind. It is assumed that the order to visit the locations is known, eliminating the computational complexity experienced in the travelling salesman problem. The path planning approach is similar to the past work by others [25], although this work accounts for the influence of the wind. The path planning algorithm produces a reference path that is supplied to the control algorithm. The control approach uses a novel sliding surface controller based on spatial dynamics of the vehicle. This approach is most similar to that of [18].

II. PROBLEM STATEMENT

The specific problem being considered is how to robustly navigate a constant velocity aircraft with a bounded turning rate from an initial position and orientation in the two dimensional plane through an ordered set of n points, (x_i, y_i) in the presence of an approximately known wind. The constant velocity assumption is made to approximate a small UAV with limited velocity control, although it also applies to any aircraft travelling at its maximum speed.

A. Vehicle Kinematic Model

The vehicle dynamics are represented by the kinematic model shown below. The control input of the aircraft, u, is the turning rate, $\dot{\psi}$, which is assumed to be bounded $|\dot{\psi}| < \dot{\psi}_{max}$. V_a is the constant velocity of the aircraft, and V_W is the velocity of the near constant wind, with $V_a > |V_W|$ at all time instances.

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} \qquad \dot{\mathbf{x}} = \begin{bmatrix} V_a \cos\psi + V_{Wx} \\ V_a \sin\psi + V_{Wy} \\ u \end{bmatrix} \tag{1}$$

B. Wind Model

It is assumed that the vehicle has access to an estimate of the wind, $\hat{\mathbf{V}}_\mathbf{W}$. The actual wind, which can be time varying, is equal to the wind estimate plus an unknown component, $\mathbf{\Delta_W}$:

$$\mathbf{V_W} = \begin{pmatrix} V_{Wx} \\ V_{Wy} \end{pmatrix} = \begin{pmatrix} \hat{V}_{Wx} + \Delta_{Wx} \\ \hat{V}_{Wy} + \Delta_{Wy} \end{pmatrix} = \hat{\mathbf{V}}_\mathbf{W} + \mathbf{\Delta_W} \tag{2}$$

Although the wind is not exactly known, it is assumed that both the unknown component of the wind and the time derivative of the unknown component are both bounded, and that the bounds are known:

$$\begin{aligned} |\mathbf{\Delta_w}|_{max} &< \beta \\ |\dot{\mathbf{\Delta}}_\mathbf{w}|_{max} &< \gamma \end{aligned} \tag{3}$$

where β and γ are known positive constants.

III. PATH PLANNING

The objective of the path planning portion of the algorithm is to define a spatial trajectory through each waypoint that the aircraft can robustly track in the presence of the unknown wind disturbances, $\mathbf{\Delta_w}$. In order to account for the presence of the unknown components of the wind and any unmodelled dynamics, a turn-rate value, $\dot{\psi}_{plan}$, less than the actual maximum turn rate, $\dot{\psi}_{max}$, is used for this optimization problem:

$$\dot{\psi}_{plan} = \alpha \dot{\psi}_{max} \tag{4}$$

where $\alpha < 1$. It is then assumed that the wind is exactly known ($\Delta_{Wi} = 0$). The optimal time path in the presence of a constant wind ($\hat{V}_{Wx}, \hat{V}_{Wy}$) with a turn-rate constraint, $\dot{\psi}_{plan}$, is then found. Since the path is calculated using this relaxed turning rate constraint, the aircraft has additional control authority to track the path in the presence of uncertainty.

A. Minimum Path Between Consecutive Points

Before addressing the problem of finding the minimum time path through several waypoints, we first address the problem of finding the minimum time path between two consecutive points $\mathbf{x_i} = (x_i, y_i, \psi_i)$ and $\mathbf{x_{i+1}} = (x_{i+1}, y_{i+1}, \psi_{i+1})$. In our previous work [19], we showed that this problem of finding an optimal path in the presence of a constant wind can be re-expressed as one without wind where the second position is treated as a virtual moving target. The velocity of this virtual target is equal and opposite to the velocity of the wind. By expressing the problem in this way, ignoring the unknown component of the wind, the equations of motion for the aircraft become:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} \qquad \dot{\mathbf{x}} = \begin{bmatrix} V_a \cos\psi \\ V_a \sin\psi \\ u \end{bmatrix} \tag{5}$$

where $\mathbf{x}(0) = \mathbf{x}_i$. The state of the virtual target, $\mathbf{x_d}$, can be expressed as:

$$\mathbf{x}_d(t) = \begin{bmatrix} x_{i+1} - \hat{V}_{Wx} t \\ y_{i+1} - \hat{V}_{Wy} t \\ \psi_{i+1} \end{bmatrix} \tag{6}$$

The goal of this redefined problem is to find the minimum time, T, such that $\mathbf{x}(T) = \mathbf{x}_d(T)$. It was also shown in [19], that the optimal aircraft path must be one of eight possible types: $RSR, LSL, RSL, LSR, LRL_{outer}, RLR_{outer}, LRL_{inner}$, and RLR_{inner}. For these eight path types, R denotes a maximum rate right turn, L denotes a maximum rate left turn, and S denotes a straight line. The *outer* subscript indicates that the angle of the center curve is greater than π, while the *inner* subscript indicates that the angle of the center curve is less than π. Unlike the well known set of Dubins paths [11], the *inner* paths are never optimal in the case of no wind. For applications where the distance between waypoints is large compared to the turning radius of the vehicle, the optimal path will consist of types RSR, LSL, RSL, and LSR.

Since the virtual target is travelling in a straight line, any point on this line can be represented by a single value, d, which is the distance from that point to the initial position of the moving target, $\mathbf{x_{i+1}}$. In order to find the smallest value of d at which the aircraft can intercept the virtual moving target (equivalent to finding the smallest time of intercept, T) a function $G_i(d)$ can be defined for each of the eight path types listed above. Each function is equal to the difference in time required by the aircraft, $T_{ai}(d)$, to travel to the point d at the given orientation using the given path type, and the time required by the virtual target to travel to that point, $T_{vt}(d)$:

$$G_i(d) \stackrel{\text{def}}{=} T_{ai}(d) - T_{vt}(d) \tag{7}$$

Since not every path type is possible for every initial and final state of the aircraft, the functions $G_i(d)$ are not defined for all values of d, although they are piecewise continuous where they exist. The minimum time path to intercept the virtual moving target can be found by calculating the points where each $G_i(d)$ function equals zero. The smallest of these values corresponds to the optimal interception of the moving target, and subsequently the optimal path to travel to the desired position and orientation in the presence of a constant wind. A schematic of the use of the virtual moving target approach is illustrated in Fig. 1.

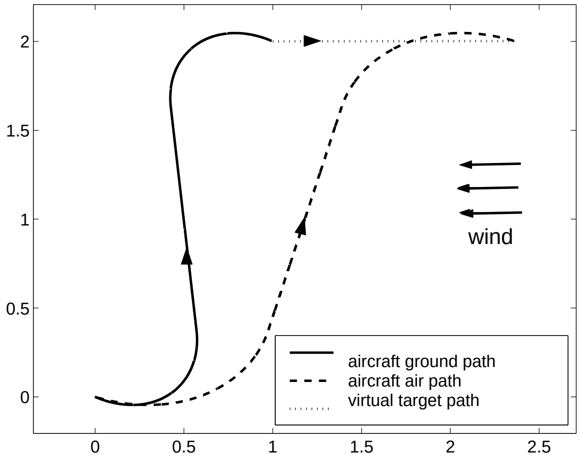

Fig. 1. Finding optimal path in wind using virtual target approach

B. Minimizing Total Path Length

Given the ability to find the optimal path between two points, the more complicated problem of finding the optimal path through several points can be addressed. The cost function for this optimization problem, J, is the total time to travel from the initial position and orientation, x_0, through n target waypoints. This total time can be expressed as the sum of the times required to travel between each set of consecutive waypoints:

$$J = \sum_{i=1}^{n} J_i \tag{8}$$

where J_i is the time required to travel from waypoint $i-1$ to waypoint i on the optimal path. If the orientation angle, ψ_i, is fixed at each waypoint, the J_i values can each be calculated using the strategy described in the previous subsection. This allows each J_i value to be expressed a function of the orientation angles ψ_{i-1} and ψ_i. Thus, as shown in [25], which explored path planning in the absence of wind, the time of total path time can be found by optimizing over the orientation angle at each waypoint:

$$J = \min_{\boldsymbol{\psi}} \sum_{i=1}^{n} J_i(\psi_{i-1}, \psi_i) \tag{9}$$

where $\boldsymbol{\psi}$ is the vector of orientation angles at each waypoint:

$$\boldsymbol{\psi} = (\psi_1, \psi_2, ..., \psi_n) \tag{10}$$

This problem can be solved using a variety of existing gradient descent optimization techniques. This work uses the FMINSEARCH function in MATLAB, which utilizes the simplex method [17]. Since the cost function in Eq. 9 is highly nonlinear, it is possible for the optimization to result in a local minimum. It was shown in [25] that most of these local minimum result from an orientation angle converging at a value near π from the actual minimum value. This results in turn angles greater than π at that point. They addressed this problem by flipping the orientation angle at any point with a turn angle greater than π and rerunning the optimization to check for improvement. During this study, it was also found that this problem could be largely avoided by the choice of the initial guess, $\hat{\boldsymbol{\psi}}$, for the nonlinear optimization:

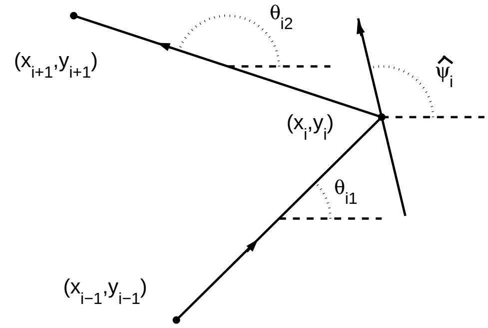

Fig. 2. Choice of initial angle for nonlinear optimization.

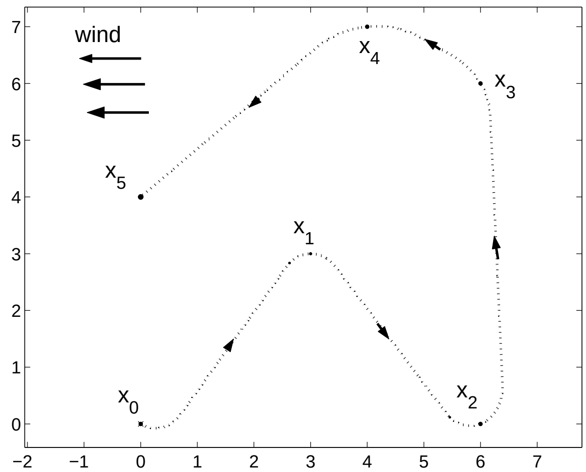

Fig. 3. Example of ground path through multiple waypoints.

$$\begin{gathered} \hat{\psi}_i = \theta_{i1} + \tfrac{1}{2}\Delta\theta_i \\ \theta_{i1} = atan2\left(y_i - y_{i-1}, x_i - x_{i-1}\right) \\ \theta_{i2} = atan2\left(y_{i+1} - y_i, x_{i+1} - x_i\right) \\ \Delta\theta_i = \theta_{i2} - \theta_{i1}, \Delta\theta_i \in [-\pi, \pi) \end{gathered} \tag{11}$$

This angle choice is illustrated in Fig. 2.

After the optimization in Eq. 9 is performed, the optimal maneuvers found using the method in Sec. III-A can be simulated in the presence of the known constant wind, $(\hat{V}_{Wx}, \hat{V}_{Wy})$, to produce a target ground path which is in the form of many closely spaced points and the orientations at those points. An example schematic of a ground path is illustrated in Fig. 3.

IV. CONTROL

The output of the path planning algorithm is an ideal path that would allow the aircraft to travel through the desired waypoints using constant rate turns and straight lines assuming that the wind is exactly known and constant, and that the kinematic model is an exact representation of the aircraft dynamics. In reality, the wind will not be exactly

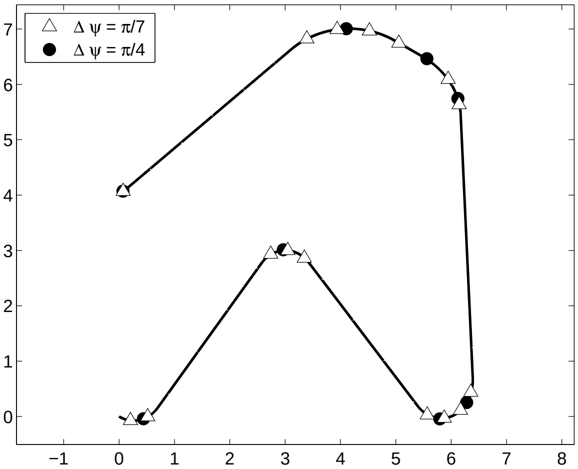

Fig. 4. Segmentation of target path.

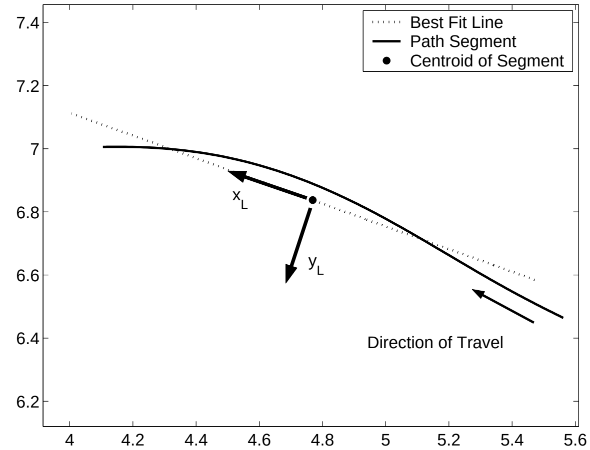

Fig. 5. Local coordinate system.

known, and there will be unmodelled vehicle dynamics. Since the ideal path was calculated assuming a turning rate smaller than the actual maximum turning rate, the aircraft has extra control authority to track the desired ground path.

A. Segmentation of path into local polynomials

In order to devise a control law to track the desired path outputted by the path planning algorithm, an analytical function describing the path must first be produced. The chosen approach is to break the larger path into shorter path segments that can each be fitted with a polynomial. These path segments are chosen so that the maximum change in the orientation angle, $\Delta\psi$, is bounded on each segment. Since the output of the path planning algorithm is a set of path points, (x_p, y_p, ψ_p), this can be achieved by growing each segment until the difference between the maximum and minimum orientations on that segment is equal to the bound, $\Delta\psi$. A new segment is then started and the process is repeated until the end of the total path is reached. This segmentation of a path is illustrated in Fig. 4, which illustrates the points where a larger path is segmented for various values of $\Delta\psi$.

Once the segments are created, a best fit line is found for the set of path points in each segment. Since the path is smooth, and the orientation along the path is bounded, there are no outliers when fitting this line, and the errors are relatively small. Thus, standard least squares can be used to fit the line. This line is used to define a rotation angle, θ_R, where the direction of the angle is along the direction of motion of the aircraft. Further, an offset position, $(\bar{x}_p, \bar{y}_p)$ is defined as the centroid of the points in the path segment. The coordinates of the path points are then mapped into a local coordinate system, (x_L, y_L), centered at $(\bar{x}_p, \bar{y}_p)$ with the local x axis aligned with θ_R. This line fitting process is illustrated in Fig. 5. The coordinates in this local system can be calculated as:

$$\begin{bmatrix} x_L \\ y_L \end{bmatrix} = \begin{bmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \bar{x}_p \\ \bar{y}_p \end{bmatrix} \right) \tag{12}$$

After rotating the points to the local coordinate system, a polynomial of degree n_p is fit to the segment points using least squares. This polynomial is used to defined the local desired position, y_{Ld}:

$$y_{Ld} = a_0 + a_1 x_L + a_2 x_L^2 + ... + a_{n_p} x_L^{np} \tag{13}$$

B. Spatial Sliding Surface Control

In order to track the individual polynomial path segments calculated above, a spatial sliding surface controller is used. This type of controller is a variation on time based sliding surface controllers which uses spatially defined dynamics instead of time based dynamics in order to avoid singularities in the control law. Since the desired paths are redefined in a local coordinate system, the local position must be calculated using Eq. 12. The orientation and wind estimates must also be transformed into the local coordinate system:

$$\begin{gathered} \psi_L = \psi - \theta_R \\ \begin{bmatrix} \hat{V}_{WLx} \\ \hat{V}_{WLy} \end{bmatrix} = \begin{bmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{bmatrix} \begin{bmatrix} \hat{V}_{Wx} \\ \hat{V}_{Wy} \end{bmatrix} \end{gathered} \tag{14}$$

The first step in the design of the controller is the definition of the spatial dynamics, assuming a constant wind, with respect to the local position variable, x_L, instead of time:

$$\frac{d}{dx_L} \begin{bmatrix} x_L \\ y_L \\ \psi_L \end{bmatrix} = \begin{bmatrix} 1 \\ (V_a \sin\psi_L + \hat{V}_{WLy})/(V_a \cos\psi_L + \hat{V}_{WLx}) \\ u/(V_a \cos\psi_L + \hat{V}_{WLx}) \end{bmatrix} \tag{15}$$

Since the polynomials that will be tracked are designed to be shallow, the perpendicular distance to the target trajectory is approximately equal to the difference between the y_L position of the vehicle and the calculated $y_{Ld}(x)$ value. Thus, the tracking error, e is defined as:

$$e = y_L - y_{Ld}(x) \tag{16}$$

A spatial sliding surface, s is then defined as:

$$s = \frac{de}{dx_L} + \lambda e \tag{17}$$

where λ is a positive constant. Unlike time based sliding surfaces, this surface is defined using spatial derivatives. The goal of the sliding surface controller is now to drive the value of s to zero. If this is achieved, the dynamics of Eq. 17 will result in the exponential decay of the error:

$$e(x_L) = e(x_0)\exp\left[-\lambda(x_L - x_0)\right] \tag{18}$$

By defining the sliding surface using the proper order, the derivative of the sliding surface with respect to x_L is an explicit function of the control input, u:

$$\frac{ds}{dx_L} = \Gamma_1 u - \frac{d^2 y_{Ld}}{dx_L^2} + \lambda\left(\frac{V_a \sin\psi_L + \hat{V}_{WLy}}{V_a \cos\psi_L + \hat{V}_{WLx}} - \frac{dy_{Ld}}{dx_L}\right) \tag{19}$$

where

$$\Gamma_1 = \frac{V_a^2 + V_a(\hat{V}_{WLx}\cos\psi_L + \hat{V}_{WLy}\sin\psi_L)}{(V_a\cos\psi_L + \hat{V}_{WLx})^2} \tag{20}$$

Thus we can chose u to satisfy:

$$\frac{ds}{dx_L} = -Ks \tag{21}$$

where K is a positive constant. This smooth control law will drive the sliding surface s exactly to zero in the absence of uncertainties or to a small boundary layer around zero in the presence of uncertainties. The final control law is:

$$u = \left[\frac{d^2 y_{Ld}}{dx_L^2} - \lambda\left(\frac{V_a \sin\psi_L + \hat{V}_{WLy}}{V_a \cos\psi_L + \hat{V}_{WLx}} - \frac{dy_{Ld}}{dx_L}\right) - Ks\right]\Gamma_2 \tag{22}$$

where

$$\Gamma_2 = \frac{(V_a\cos\psi_L + \hat{V}_{WLx})^3}{V_a^2 + V_a(\hat{V}_{WLx}\cos\psi_L + \hat{V}_{WLy}\sin\psi_L)} \tag{23}$$

This control law requires that the first and second spatial derivatives of the desired path be known. Since all of the desired paths are fitted with polynomial functions, shown in Eq. 13, these values can be easily calculated:

$$\frac{dy_{Ld}}{dx_L} = a_1 + 2a_2 x_L + 3a_3 x_L^2 + ... + n_p a_{n_p} x_L^{n_p - 1} \tag{24}$$

$$\frac{d^2 y_{Ld}}{dx_L^2} = 2a_2 + 6a_3 x_L + ... + n_p(n_p - 1)a_{n_p} x_L^{n_p - 2} \tag{25}$$

TABLE I

SIMULATION PARAMETERS.

Parameter	Value
V_a	1 units/sec
$\dot{\psi}_{max}$	1.5 rad/sec
$\hat{V}_{Wx}$	-0.3 units/sec
$\hat{V}_{Wy}$	0 units/sec
$\Delta_{W1}, \Delta_{W2}, \Delta_{W3}, \Delta_{W4}$	0.05 units/sec
ω_{W1}, ω_{W2}	2π rad
ϕ_W	$\pi/2$ rad
$\dot{\psi}_{plan}$	1 rad/sec
$\Delta\psi$	$\pi/4$ rad
n_p	7
K	30
λ	10

V. SIMULATION RESULTS

In order to evaluate the path planning and control algorithms, the complete system was tested using simulation. The unknown wind component was modelled as the sum of constants and sinusoids with unknown magnitudes and frequencies:

$$\begin{bmatrix} \Delta_{Wx} \\ \Delta_{Wy} \end{bmatrix} = \begin{bmatrix} \Delta_{W1} + \Delta_{W2}\sin(\omega_{W1}t) \\ \Delta_{W3} + \Delta_{W4}\sin(\omega_{W2}t + \phi_W) \end{bmatrix} \tag{26}$$

The performance specifications and wind values used in this simulation are listed in Table I. These values correspond to a constant wind value which is roughly 25% the velocity of the vehicle, a 17% error in the estimate of the constant wind magnitude, and wind gusts with an amplitude of 24% the velocity of the constant wind. More rigorous theoretical bounds on the tracking error can be calculated from the maximum uncertainty values in Eq. 3, using standard techniques [24]. Despite these disturbances, the aircraft distance from the desired path remains within less than 6% of the minimum turning radius of the vehicle. The simulated path of the vehicle is shown in Fig. 6. The performance of the controller, including the tracking error, sliding surface value, and controller input are illustrated in Fig. 7. The discontinuities in the controller inputs result from the switching between local controllers for the various path segments.

VI. CONCLUSIONS AND FUTURE WORK

A complete system including both path planning and control algorithms for the surveillance of multiple locations in the presence of wind has been presented. The path planning algorithm calculates the shortest time path through all of the waypoints using a turning rate which is less than the actual maximum turning rate. This algorithm produces a ground path that can be tracked by the control algorithm. The control algorithm breaks the desired trajectory into smaller sections which can each be approximated by a polynomial. A spatial sliding surface controller is then used to track each polynomial in the presence of the unknown wind disturbances. The complete system has been demonstrated

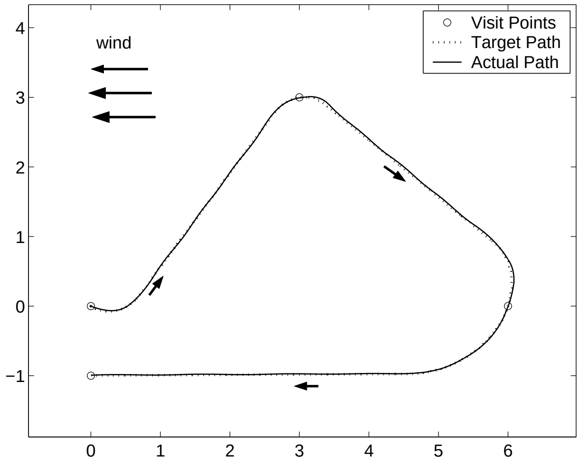

Fig. 6. Simulation results.

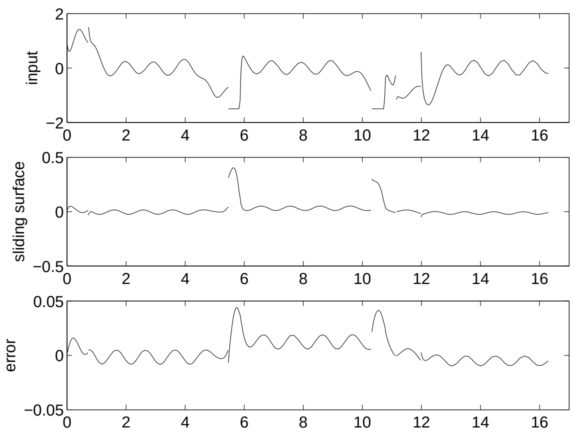

Fig. 7. Controller performance.

using simulation and shown to be robust against uncertainties in the wind estimate.

The next major step in this research is the flight testing of the proposed algorithms on one of our experimental fixed-wing aircraft. Each aircraft flies under the combined control of an off-the-shelf Piccolo avionics package and an onboard computer. The Piccolo performs low level control and provides GPS measurements and wind estimates. The onboard computer runs higher level algorithms and can provide the Piccolo with turn rate commands. These aircraft travel at a nominal speed of 20 m/s, and wind speeds of over 5 m/s have been encountered during flight testing. Further theoretical work could include the inclusion of variable vehicle speed, calculation of optimal waypoint ordering, and dynamic updating of the path in the presence of changes in the wind estimate or waypoints.

References

[1] P. K. Agarwal, P. Raghavan, and H. Tamakai, "Motion planning for a steering-constrained robot through moderate obstacles," in *Proc. of the 27th Annual ACM Symposium on the Theory of Computing*, 1995.

[2] R. W. Beard, T. W. McLain, and M. Goodrich, "Coordinated target assignment and intercept for unmanned air vehicles," in *Proc. of the 2002 IEEE International Conf. on Robotics and Automation*, 2002.

[3] A. Bicchi, G. Casalino, and C. Santilli, "Planning shortest bounded-curvature paths for a class of nonholonomic vehicles among obstacles," *Journal of Intelligent and Robotic Systems*, vol. 16, 1996.

[4] J.-D. Boissonnat, A. Cerezo, and J. Leblond, "Shortest paths of bounded curvature in the plane," in *Proceedings of the 1992 IEEE Int. Conf. on Robotics and Automation*, 1992.

[5] X.-N. Bui, J.-D. Boissonnat, P. Soueres, and J.-P. Laumond, "Shortest path synthesis for dubins non-holonomic robot," in *Proceedings of the 1994 IEEE Int. Conf. on Robotics and Automation*, 1994.

[6] D. Chwa, "Sliding-mode tracking control of nonholonomic wheeled mobile robots in polar coordinates," *IEEE Transactions on Control Systems Technology*, vol. 12, 2004.

[7] M. Corradini and G. Orlando, "Robust tracking control of mobile robots in the presence of uncertainties in the dynamical model," *Journal of Robotic Systems*, vol. 18, 2001.

[8] D. L. Cripps, "Spatially-robust vehicle path tracking using normal error feedback," in *Proceedings of the SPIE, Unmanned Ground Vehicle Technology III*, 2001.

[9] O. Dagci, U. Ogras, and U. Ozguner, "Path following controller design using sliding mode control theory," in *Proceedings of the American Control Conference*, 2003.

[10] M. Davidson, V. Bahl, and K. L. Moore, "Spatial integration for a nonlinear path tracking control law," in *Proceedings of the 2002 American Control Conference*, 2002.

[11] L. Dubins, "On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents," *American Journal of Mathmatics*, vol. 79, 1957.

[12] M. Egerstedt, X. Hu, and A. Stotsky, "Control of mobile platforms using a virtual vehicle approach," *IEEE Transactions on Automatic Control*, vol. 46, 2001.

[13] R. Frezza, G. Picci, and S. Soatto, "A lagrangian formulation of nonholonomic path following," in *Proceedings of the Confluence of Vision and Control*, 1998.

[14] J. Guldner and V. I. Utkin, "Sliding mode control for an obstacle avoidance strategy based on an harmonic potential field," in *Proc. of the 32nd IEEE International Conf. on Decision and Control*, 1993.

[15] J. Guldner, V. I. Utkin, and J. Ackermann, "A sliding mode control approach to automatic car steering," in *Proc. of the American Control Conference*, 1994.

[16] T. Hamel, P. Soueres, and D. Meizel, "Path following with a security margin for mobile robots," *International Journal of Systems Science*, vol. 32, 2001.

[17] J. Lagarias, J. Reeds, M. Wright, and P. Wright, "Convergence properties of the nelder-mead simplex method in low dimensions," *SIAM Journal of Optimization*, vol. 9, 1998.

[18] R. Lenain, B. Thuilot, C. Cariou, and P. Martinet, "A new nonlinear control for vehicle in sliding conditions: Application to automatic guidance of farm vehicles using rtk gps," in *Proceedings of the IEEE International Conference on Robotics and Automation*, 2004.

[19] T. McGee, S. Spry, and K. Hedrick, "Optimal path planning in a constant wind with a bounded turning rate," in *Proc. of the AIAA Guidance, Navigation and Control Conference and Exhibit*, 2005.

[20] H. Mechlih, "Sliding mode path control for an autonomous vehicle," in *Proceedings of the IEEE-IEE Vehicle Navigation and Information Systems Conference*, 1993.

[21] S. Ranthinam, R. Sengupta, and S. Darbha, "Resource allocation algorithm for multi-vehicle systems with non-holonomic constraints," 2005, accepted in IEEE Trans. on Automation Science and Eng.

[22] W. Ren and R. W. Beard, "Trajectory tracking for unmanned air vehicles with velocity and heading rate constraints," *IEEE Transactions on Control Systems Technology*, vol. 12, 2004.

[23] K. Savla, E. Frazzoli, and F. Bullo", ""on the point-to-point and traveling salesperson problems for Dubins' vehicle"," in *Proc. of the American Controls Conference*, Portland, OR, June 2005, 786-791.

[24] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, New Jersey: Prentice Hall, 1991.

[25] Z. Tang and U. Ozguner, "On motion planning for multi-target surveillance with limited resources of mobile sensor agents," *IEEE Transactions on Robotics*, vol. 21, 2005.

[26] G. Yang and V. Kapila, "Optimal path planning for unmanned air vehicles with kinematic and tactical constraints," in *Proceedings of the 41st IEEE Int. Conf. on Decision and Control*, 2002.

[27] J.-M. Yang and J.-H. Kim, "Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots," *IEEE Transactions on Robotics and Automation*, vol. 15, 1999.