Guaranteed Strategies to Search for Mobile Evaders in the Plane (Draft)

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***Abstract*— This paper studies several problems of search in the two-dimensional plane for a mobile intruder or evader. In each case, both the searcher and evader are assumed to have bounded velocities, with the velocity of the searcher greater than that of the evader. The searcher has a sensor with a circular footprint which allows it to detect the evader if its distance is less than the sensor radius. Unlike much of the past work done in search theory, the proposed strategies make no assumptions about the motion of the evader, and are guaranteed to succeed for any trajectory of the evader. The three problem formulations studied include the search for mobile targets through a linear corridor, the search for or bounding of a mobile target which starts inside of a circular region, and the prevention of a mobile target from entering a circular region. The extension of the proposed strategies for multiple searchers is also discussed.**

I. INTRODUCTION

This paper studies several problems of search in the two-dimensional plane for a mobile intruder or evader. It is assumed that the searcher and the evading target have maximum velocities, V_s and V_t, with $V_s > V_t$. No restrictions are placed on the maximum turning rates of either party. The searcher is assumed to have a sensor with a circular footprint of radius R_s which will detect the target if the target is inside of the sensor footprint. Unlike much of the past work done in search theory, the proposed strategies make no assumptions about the motion of the evader, and are guaranteed to succeed for any trajectory of the evader. The problem formulations given, although abstracted, have potential application for a wide variety of practical problems including border and harbor patrol, convoy protection, and search and rescue operations. The first problem involves the search for mobile targets through a corridor bounded by parallel lines. The second problem explores how to search for or trap a mobile target whose initial position is known to be within a circle of known radius greater than the sensor radius. The final problem, which is very similar to the second, explores how to create a perimeter around a fixed point in the plane in order to prevent intruders from approaching the point without being detected. The major contribution of this paper is a strategy for searching the circular regions, which is an improvement to a similar strategy outlined in [15]. While the proof of optimal strategies is difficult for these problems, it is argued that the optimal strategy must be close to the proposed solution. This is done by reducing the search problem to the simpler problem of travelling around an expanding circle for which the optimal solution can be found. It is also shown how these strategies can be executed by teams of multiple searchers to improve performance.

There has a large amount of research into search theory dating back to WWII, including several books devoted to the topic [7], [2], [11]. Many search problems are also closely related to game theory [10]. One major group of problems involves the search for stationary or quasi-stationary targets. Baeza-Yates outlined optimal methods for searching for stationary points or lines in planar grids [6]. Several researchers have studied optimal trajectories to completely cover an area with a sensor or tool with minimal overlap between passes [9], [1], [5], [13]. Search problems with mobile targets can be further classified by the intention and information of the target. In the rendezvous search problem, both players cooperate to find one another [3]. In the helicopter and submarine problem, first proposed by Danskin [8], a helicopter searches for an evading submarine that does not know the position of the searcher. Several other researchers have studied the same problem [18], [16], [19]. Another study examines guaranteed search strategies for an evader in an complex enclosed workspace by a searcher with a line of sight sensor [12]. More recently, search with teams of cooperating agents has become an active research topic [14], [17], [4].

II. SEARCH OF A CORRIDOR

The first planar search problem considered is the patrol of a corridor between parallel borders separated by width W. This problem was solved by Koopman [11] in the 1940s in order to determine optimal patrol strategies for aircraft searching for ships in a channel. It is reviewed here since the concepts used to solve this problem are later shown to be applicable to more difficult search problems in the plane. The corridor search problem assumes that at the initial time, the set of all possible positions of the target, which we shall refer to as the target set, is the area to one side of a line perpendicular to the borders of the corridor. The searcher begins on one of the corridor borders with its circular sensor tangent to the boundary of the target set as shown in Fig. 1a. Since the evader is mobile, the target set grows normal to its boundary at the maximum velocity of the evader, V_t. Thus, in order to keep its sensor tangent to the border of the target set, the searcher must travel at an angle ϕ to the normal of the target set (Fig. 1b), where ϕ is calculated as:

$$\phi = \arcsin\left(\frac{V_t}{V_s}\right) \tag{1}$$

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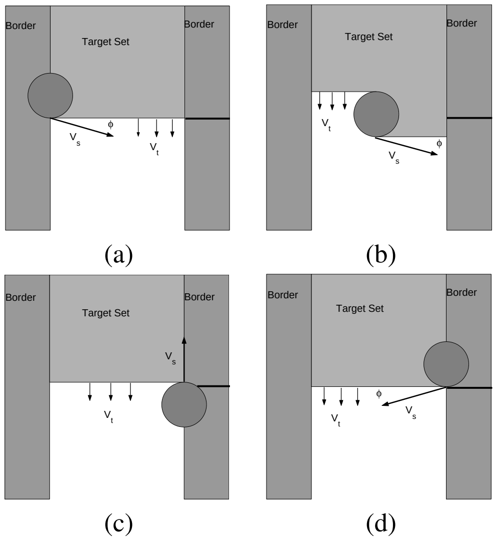


Fig. 1. Corridor Patrol Strategy.

By following this strategy, when the searcher reaches the opposite border, the boundary of the target set is still a line perpendicular to the borders, although the searcher is now outside of the target set (Fig. 1c). The searcher next travels parallel to the border until it is again over the target set, and its sensor is tangent to the boundary of the target set (Fig. 1d). The searcher then repeats the same maneuver to return to the opposite border, completing one cycle of the maneuver. The time required for the searcher to complete each half cycle, t_{s1}, is the sum of the times required to cross the corridor and the time to travel up the border.

$$t_{s1} = \frac{W}{V_s \cos(\phi)} + \frac{2R_s}{V_s + V_t} = \frac{W}{\sqrt{V_s^2 - V_t^2}} + \frac{2R_s}{V_s + V_t} \quad (2)$$

If the travel time of the searcher for each half cycle is equal to the time required for the evader to travel the width of the search sensor, the boundary of the target set at the beginning of each half cycle will remain stationary.

$$\frac{W}{\sqrt{V_s^2 - V_t^2}} + \frac{2R_s}{V_s + V_t} = \frac{2R_s}{V_t} \quad (3)$$

Koopman referred to this case as a *symmetric barrier*. Solving for the width of the corridor in this case gives the maximum width of a corridor that can be patrolled:

$$W^* = 2R_s \left(\frac{1}{V_t} - \frac{1}{V_s + V_t} \right) \sqrt{V_s^2 - V_t^2} \quad (4)$$

If the corridor width is larger than W^*, the size of the target set will grow after each cycle, and it cannot be guaranteed that the target will be detected. If the width is less than W^*, the target set will shrink, allowing it to be searched. Koopman called these cases *retreating element barriers* and *advancing element barriers* respectively.

III. CIRCULAR BARRIERS AGAINST FLEEING EVADERS

Another problem to consider is how to search for or bound an evader that starts within a known circular region. This problem could have applications for embargoes, patrolling jail perimeters, or search for mobile targets. Assuming the target is initially inside a circle of radius R_{to}, and the searcher is outside of this circle with its sensor tangent to target set, we explore how the searcher should travel to guarantee that the target set is bounded for the largest possible initial circle with radius R_{to}^*. For all circles smaller than this, the bounding strategy should also provide a method to shrink the area target set to zero in order to guarantee capture. This is analogous to the advancing barrier for the channel patrol problem.

A. Upper Bound for R_{to}^*

Since the optimal solution for the guaranteed search of an expanding disk is unknown [7], a good first step to exploring this problem is to set bounds on R_{to}^*. One such bound can be found by assuming that the searcher can apply its maximum search rate arbitrarily in the plane. For the problem under consideration, the maximum search rate is the product of the search velocity, V_s and the width of the sensor, $2R_s$. The growth rate of the target set can be calculated as the product of its perimeter, $2\pi R_{to}$, and the velocity of the target, V_t. The maximum radius of the target set for this unconstrained problem, which acts as an upper bound for our problem, can then be found by setting the search rate equal to the growth rate of the target set.

$$R_{to}^* \leq \frac{R_s V_s}{\pi V_t} \quad (5)$$

B. Circular Search Pattern

One simple strategy to bound an evader, which has been offered by several researchers [11], [16], is to travel a simple circular pattern. Both of these studies calculate the largest circle that can be bounded using a circular pattern to be:

$$R_{to}^* = R_s(V_s/\pi V_t - 1) \quad (6)$$

This equation is found by setting the time for the searcher to travel around this circle $2\pi(R_{to}^* + R_s)/V_s$ to be equal to the time it takes the evader to travel across the diameter of the search sensor $2R_s/V_t$. One limitation of this calculation, however, is that it assumes that the evader is travelling radially from the center of the circle. The optimal path of the evader is actually to travel at a slight angle, γ_{opt} to the radius of the circle as illustrated in Fig.2. This optimal angle γ_{opt}

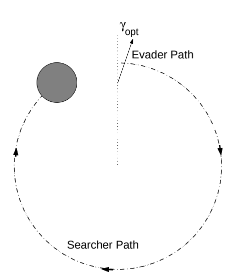


Fig. 2. Optimal Evasion Strategy Against Circular Search Path.

can be calculated by maximizing the quantity $dR_t/d\theta_{rel}$:

$$\frac{dR_t}{d\theta_{rel}} = \frac{\dot{R}_t}{\dot{\theta}_p - \dot{\theta}_t} = \frac{V_t \cos(\gamma)}{V_s/R_p - V_t \sin(\gamma)/R_t} \tag{7}$$

where R_t is the radius of the evading target, R_p is the radius of the searcher, and θ_{rel} is the relative angle between the searcher and evader. The optimal choice of γ to maximize $dR_t/d\theta_{rel}$ is:

$$\gamma_{opt} = \arcsin\left(\frac{V_t R_p}{V_s R_t}\right) \tag{8}$$

Although travelling at γ_{opt} reduces the radial velocity of the target, it forces the searcher to travel more than a full circle to catch the target. Thus if the evader follows the path specified by Eq. 8, it can escape the circle defined by Eq. 6. Although the circular search path is simple, it is not optimal, and it doesn't lend itself to a method of reducing the target set for $R_{to} < R^*_{to}$.

C. Proposed Pattern to Search Expanding Disk

The goal of the search of the expanding disk problem can be reexpressed as the reduction of the area of the target set, A_{TS}, to zero in the minimum time. Since an analytical solution to this problem is difficult because of the partial information of the evader's position, an approach is taken to develop a strategy based on insight from similar problems. Since intuition can often be wrong, the performance of the developed strategy is compared against the theoretical bound on the optimal strategy in Eq. 6. First, we explore the instantaneous rate of change of the target set area which can be calculated as the difference between the growth rate of the target set resulting from possible target motion and the reduction of the target set area by the motion of the searcher:

$$\dot{A}_{TS} = V_t P_{TS} - V_s L_{S\perp} \tag{9}$$

where P_{TS} is the perimeter of the target set and $L_{S\perp}$ is the width of the sensor footprint perpendicular to the velocity of the searcher that is over the target set. The maximum value of $L_{S\perp}$ is the width of the sensor, $2R_s$. While minimizing $\dot{A}_{TS}$ is difficult because P_{TS} and $L_{S\perp}$ are complicated functions of the chosen trajectory, several important properties for a well designed strategy can be inferred from Eq. 9.

Property 1: The well designed search strategy should travel along the perimeter of the target set.

In order to minimize the value of $\dot{A}_{TS}$, the search strategy should also minimize the perimeter of the target set, P_{TS}. If the searcher were to travel into the target set as opposed to along the perimeter, it would create a channel that would greatly increase the perimeter of the target set. This property eliminates divide and conquer type strategies from being considered for search of the circle.

Property 2: The target set should remain near circular when the well designed search strategy is applied.

For any given planar area, the shape enclosing that area with the smallest perimeter is the circle. Thus search strategies that preserve the circular shape of the target set will be superior to those that do not. This property eliminates strategies such as line sweeps from being considered.

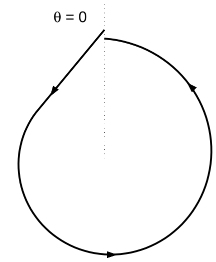


Fig. 3. Minimum length path around expanding circle.

Property 3: The outer edge of the search sensor should be tangent to the boundary of the target set during most of the well designed search.

From Property 1, it was found that the searcher should travel along the perimeter of the target set. If the search sensor is tangent to the boundary of the target set, $L_{S\perp}$ will equal its maximum value.

In order to develop a search strategy which accounts for the above properties, we consider a simplified problem. Consider a mobile agent, travelling at a constant speed, V_s, that must travel around an expanding circle in the plane. The radius of the circle expands linearly in time at a rate of V_t. At the initial time, the agent is at $\theta = 0$, and its distance from the center of the circle is greater than the initial radius of the circle. The minimum length (and thus time) path that the agent can take to travel around the expanding circle is to first travel a straight line that will intersect the expanding circle tangentially and then to travel along the perimeter of the expanding circle. This path is illustrated in Fig. 3. The solution to this simplified problem can be slightly modified to produce the improved search method, illustrated in Fig. 4. This search strategy consists of alternating straight lines and outward spiral maneuvers of the searcher, which result from a single rule that governs the velocity direction of the searcher. At the initial time, the searcher begins outside of the target set with its sensor radius tangent to the circular target set. From this point forward, at each instance in time, a line is drawn which is tangent to both the search sensor and the outside of the target set, as shown by the dotted line in Fig. 4a. The searcher then travels at an angle ϕ to this line, which is the same lead angle calculated in Eq. 1. During the first portion of the search, this produces a linear motion of the searcher. Once the outside of the sensor footprint becomes tangent to the circular portion of the target set (Fig. 4b), the tangent line to the search sensor and the target set is tangent to both at the same point. As the searcher continues to travel at a lead angle ϕ to this tangent line, it will travel along an outward spiral such that the radial velocity of the searcher is equal to the target velocity:

$$\dot{R}_p = V_s \sin(\phi) = V_t \tag{10}$$

$$\dot{\theta}_p = \frac{V_s \cos(\phi)}{R_p} = \frac{\sqrt{V_s^2 - V_t^2}}{R_p} \tag{11}$$

where R_p and θ_p are the radius and angle of the searcher with respect to the center of the target set.

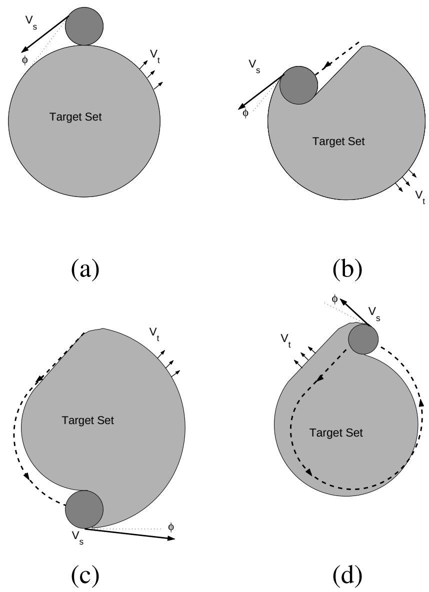


Fig. 4. Proposed pattern.

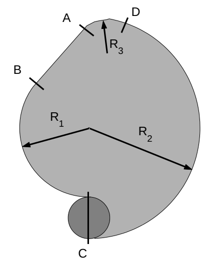


Fig. 5. Sections of target set.

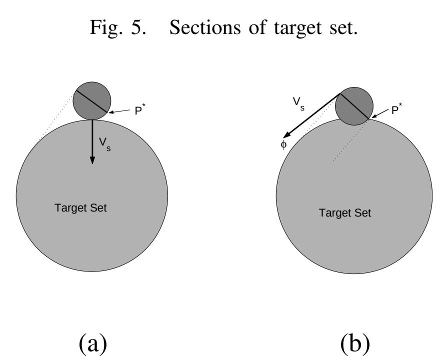


Fig. 6. Marginal improvement to strategy.

The searcher continues this spiral maneuver (Fig. 4c) until it reaches the small arc which connects the circular and linear portions of the target set (Fig. 4d). At this point, the search travels along a smaller outward spiral using the same lead angle strategy. Following the search strategy described above will produce a target set whose perimeter can be divided into four sections as shown in Fig. 5. The first section, A-B, is a linear section. The second section of the perimeter, B-C, defines the arc of a circle whose radius, (R_1), is equal to the radius of the searcher minus the sensor radius. The third section of the perimeter, C-D, defines the arc of a circle whose radius, (R_2), is equal to the radius of the searcher plus the sensor radius. The final section of the perimeter, D-A, is the arc of a smaller circle which connects the linear section, A-B, and the larger circle, C-D. During the initial stages of a search, while the target set is large enough, the radius of the D-A section, R_3, will be greater than the radius of the search sensor, R_s, when the searcher reaches this section of the perimeter. During these cases, the outside of the search sensor will always be tangent to the edge of the target set and the search path will be differentiable. When the target set gets smaller however, the search sensor will completely cover the D-A section of the perimeter. In these cases, the rule described above will produce a sharp change in the direction of the searcher. When this condition occurs, there is a modification to the search strategy that can slightly improve performance. The diameter of the sensor which is perpendicular the line tangent to the sensor and target set can be used to define the point P^* as illustrated in Fig. 6a. Whenever this point P^* is outside the target set, the searcher can improve performance by travelling radially inward toward the center of the target set until P^* touches the target set (Fig. 6b). This modification can also be used at the beginning of the search effort (Fig. 4a). Although this modification can provide a small improvement, it complicates the motion and will not considered when calculating the performance of the proposed method.

In order to determine the largest initial radius of the target set that can be bounded, R^*_{to}, using the proposed pattern, it assumed that R_3 in Fig. 5 is equal to zero. This conservative assumption results in a slightly smaller R^*_{to} than is actually achievable, although it greatly simplifies the calculation. The path of the searcher can now be divided into two portions: the straight section and the outward spiral. Consider the time, t_{s2}, required by the searcher to travel the straight portion of the search, assuming the searcher starts at $R_{to} + R_s$. This time can be calculated as:

$$t_{s2} = \frac{2\sqrt{R_{to} R_s}}{V_s \cos\phi} = 2\sqrt{\frac{R_{to} R_s}{V_s^2 - V_t^2}} \tag{12}$$

This time is the ratio of the distance $2\sqrt{R_{to} R_s}$, shown in Fig. 7, to the velocity of the searcher parallel to this line segment. The radius of the searcher immediately after travelling the straight maneuver, R_{p2}, can be calculated:

$$R_{p2} = R_{to} - R_s + V_t t_{s2} \tag{13}$$

The radius of the searcher after the spiral maneuver can then be calculated by integrating Eqs. 10 and 11 :

$$R_p(\theta_f) = R_{p2} \exp\left(\frac{(\theta_f - \beta) V_t}{\sqrt{V_s^2 - V_t^2}}\right) \tag{14}$$

where $\theta_f = 2\pi$ in the case of a single searcher, and β, illustrated in Fig 7, is the angle of the target set traversed during the straight maneuver:

$$\beta = \cos^{-1}\left(\frac{R_{to} - R_s}{R_{to} + R_s}\right) \tag{15}$$

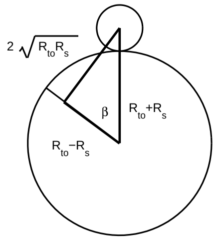


Fig. 7. Calculating length of straight section.

The radius of the largest boundable circle, using the proposed spiral search can then be calculated by enforcing that the searcher finishes a cycle of the search where it started:

$$R_s(\theta_f) = R^*_{to} + R_s. \tag{16}$$

Combining Eqs. 12 - 16 yields an expression that can be solved for R^*_{to}:

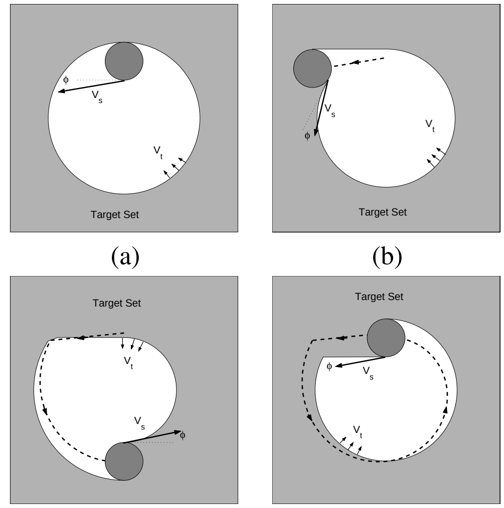
$$R^*_{to} + R_s = \left(R^*_{to} - R_s + 2V_t\sqrt{\frac{R^*_{to}R_s}{V_s^2 - V_t^2}}\right) * \exp\left(\frac{V_t\left(\theta_f - \cos^{-1}\left((R^*_{to} - R_s)/(R^*_{to} + R_s)\right)\right)}{\sqrt{V_s^2 - V_t^2}}\right) \tag{17}$$

For $V_s = 20$, $V_t = 1$, and $R_s = 100$, a circular pattern can bound $R^*_{to} = 536$, while the proposed strategy can bound $R^*_{to} = 625$. The theoretical upper bound from Eq. 6 is 636.

IV. CIRCULAR BARRIERS AGAINST INTRUDERS

The third problem considered is how to prevent an intruder from entering a circular region without being detected. This problem is essentially the reverse of the circular barrier problem described above. Assuming the target is initially outside the circle R_{to}, and the searcher is inside of this circle with its sensor tangent to R_{to}, we explore how the searcher should travel to guarantee that the intruders's minimum distance from the center of the circle is bounded. The goal is to find a strategy to guarantee bounding for the largest initial circle R^*_{to}. For all circles smaller than this, the bounding strategy should also provide a method to expand the cleared area to its maximum.

The strategy of Section III can be reversed to expand the cleared area. This is done by defining the lead angle ϕ toward the center of the cleared region, which produces inward spirals rather than outward spirals. This strategy is illustrated in Fig .8. The application of this strategy will result in a stable final trajectory, analogous to a stable limit cycle, around the cleared area. For large initial radii of the cleared area, R_{to}, the cleared area will shrink since the searcher cannot clear the perimeter fast enough. For small initial radii of the cleared area, the cleared area will be enlarged since the searcher can clear area faster than the possible invader motion fills it in. This existence of a stable final trajectory is different from the unstable trajectories of III. When searching for a target on the inside of an expanding circular region, if R_{to} is larger than R^*_{to}, the target set will



(a) (b) (c) (d)

Fig. 8. Proposed pattern for intruders.

increase indefinitely, while if R_{to} is smaller than R^*_{to}, the target set will shrink to zero area. This is analogous to an unstable limit cycle.

V. MULTIPLE SEARCHERS

The strategies described above can be easily extended to multiple searchers. For the search of the corridor [11], multiple searchers can travel abreast, effectively increasing the width of the search sensor. Unlike the corridor case, for the search of the expanding or collapsing circular regions, multiple searchers cannot travel abreast unless they are able to vary their velocities so that their angular velocities around the perimeter are equal. Considering the search of the expanding circle for multiple searchers, Eq. 9 becomes:

$$\dot{A_{TS}} = V_t P_{TS} - \sum_{i=1}^{n} V_s L_{S\perp i} \tag{18}$$

where, n is the number of searchers. This suggests that the same three properties outlined in Section III for a single search apply to the multiple searcher case. If the multiple searchers are equally spaced around the perimeter of the target set, the simplified problem of traversing an expanding circle for a single searcher shown in Fig. 3 can be modified to find the optimal path for a single searcher to travel from $\theta = 0$ with respect to the expanding circle to $\theta = 2\pi/n$. Thus, the same control law from the single vehicle case is directly applicable to the multiple vehicle case. This strategy will produce a target set shape similar to Fig. 5, only instead of a single straight line section, there will be a straight line for each searcher. This is illustrated in Fig. 9 which illustrates three searchers in the expanding circle case. In order to calculate R^*_{to} for multiple searchers, Eq. 14 can be modified by setting $\theta_f = 2\pi/n$. This imposes the constraint that each searcher will finish the cycle where the searcher in front of it started. In order to further study the effects of

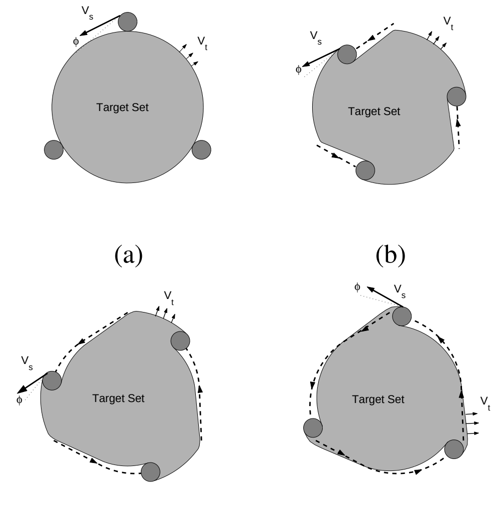


Fig. 9. Search with multiple searchers.

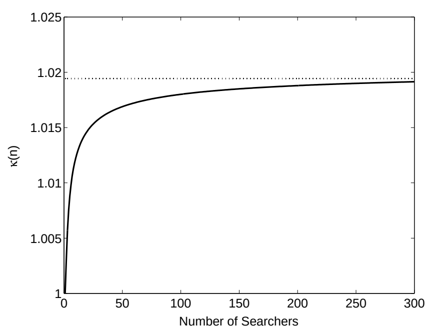


Fig. 10. Cooperation gain, κ for multiple searchers.

cooperating searchers, we define a cooperation gain, $\kappa(n)$:

$$\kappa(n) = \frac{R^*_{to}(n)}{nR^*_{to}(1)} \tag{19}$$

where $R^*_{to}(n)$ is the maximum boundable radius with n searchers.

Figure 10 illustrates, that $\kappa(n)$ is monotonically increasing signifying that n searchers can bound a circle with a radius greater than n times the radius that can be bound by a single searcher. Further, as n increases, $\kappa(n)$ approaches the ratio of the theoretical upper bound in Eq. 6 to $R^*_{to}(1)$:

$$\lim_{n\to\infty} \kappa(n) = \frac{R_s V_s}{\pi V_t}[R^*_{to}(1)]^{-1} \tag{20}$$

Thus, as the number of searchers increases,the maximum boundable radius approaches the theoretical upper bound. It should also be noted that since the radius of the maximum boundable circle is roughly proportional to the number of searchers, the maximum boundable area is roughly proportional to the square of the number of searchers.

VI. CONCLUSIONS

Guaranteed strategies for several problems of search in the two-dimensional plane for a mobile intruder or evader have been presented. The proposed strategies make no assumptions about the motion of the evader, and guarantee that the searcher will travel within sensor range of the evader for any trajectory of the evader. The three problem formulations studied include the search for mobile targets through a channel bounded by parallel lines, the search for or bounding of a mobile target which starts inside of a circular region, and the prevention of a mobile target from entering a circular region. While no optimality proof for search of the circular regions exist, the proposed strategies are shown to perform close to a theoretical bound on the problem. The application of the strategy for multiple searchers is also presented.

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